2018-06-18 STiM
Electronics and telecommunication, summer semester of the academic year 2017/18
Selected topics in mathematics
Exam problems (2018-06-18, 8:00, BM-324)

1. Axiomatic structure of mathematics, illustrate it with a) Euclidean, hyperbolic and elliptic geometries, b) standard and Lorenz addition of speeds, c) Bertrand paradox
2. A polynomial collocation (also in Stevin basis, Lagrange basis)
3. A trigonometric collocations: full/complete/standard one, cosine one, Hartley one
4. Collocations in exponential basis $(\exp(kt))_{k=0,1,\ldots,n}$ and in Euler basis $(\exp(ikt))_{k=-m,\ldots,m-1,m}$, its relation to standard trigonometric collocation
5. The bit reverse, the butterfly glue and an idea of FFT
6. The least-square fit (aka the method of least squares)
7. An eigenpair of a matrix, its geometrical (via M-images), mechanical and computational interpretations
8. The theorem on how many elements sit in the spectrum of a matrix (incl. the proof)
9. An equivalence relation, examples of it (incl. the similarity of matrices)
10. Theorems on characteristic polynomials of similar matrices, and on spectra of similar matrices (incl. proofs); provide an example that the inverse theorem does not hold true
11. The theorem on the diagonalisation of a matrix having the full spectrum (incl. the proof)
12. The Cayley-Hamilton theorem and its consequence, in particular concerning the matrix exponential
13. The Lagrange-Sylvester theorem on $f(M)$, where f is a (complex) function, M is a square matrix
14. The ODE1 describing the exponential growth, the discharge of a capacitor
15. The Newton cooling problem
16. The logistic/Verhulst equation
17. HLODE2CC (homog.lin.ordin.differential eqn of order 2 with const.coeffs), incl. that describing the movement of a mass on a spring, the flow of an electric current along (both serial and paralel) circuit RLC
18. A system of HLODE1CC and its solution via a matrix exponential $\exp(Mt)$
19. Four families of classical orthogonal polynomials
20. A LPDE2 in two (and more) variables and lits their canonical forms
21. The spectral recognition of the type of a LPDE2
22. The telegraphists' equation
23. A magma/grupoid, semigroup, monoid and group; give examples of these binary algebras
24. A symmetric group S_n and its subgroups, in particular the dihedral group D_n and its instances (via permutations, permutation matrices, rigid moves of a regular n-gon)
25. A cyclic structure of a permutation
26. The group $\mathrm{GL}(n,\boldsymbol{R})$ and its subgroups $\mathrm{O}(n)$ and $\mathrm{SO}(n)$, as well as $\mathrm{GL}(n,\boldsymbol{C})$, $\mathrm{U}(n)$ and $\mathrm{SU}(n)$
27. A simple group, the theorem on the simplicity of $(\boldsymbol{Z_n},+)$
28. A homomorphism, epimorphism (or: onto), endomorphism (or: 1-1), isomorphism, automorphism; examples of these transformations
29. A ring, a commutative/Abelian ring, and a field (incl. $\mathrm{GF}(n)$); examples
30. A linear/vector space, V, over a field, S; examples (incl. the space of solutions to an ODEn)
31. A linear combination, as well as a linear independence, of elements of a linear space. The matrix criterion on the linear independence
32. A metric and a metric space; examples
33. A norm; examples of norms of vectors, matrices, functions
34. An inner/scalar product of vectors and that of functions
35. An unitary space (i.e., with an inner product), Hilbert space (i.e., complete metric space with a metric generated by the inner product), Banach space (i.e., complete metric generated with a metric generated by a norm)

{45 lective hours, 45 class hours}
Adam Marlewski 2018-06-14